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5. No. of Question Paper : 89
 Unique Paper Code : 32351102 I
 Name of the Paper : Algebra
 Name of the Course : B.Sc. (Hons.) Mathematics
 Semester : I

Duration : 3 Hours

Maximum Marks : 75

Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.

All questions are compulsory.

- (a) Find polar representation of the complex number : 6
 $z = \sin a + i(1 + \cos a)$, $a \in [0, 2\pi)$.
- (b) Find $|z|$ and $\arg z$, $\arg(-z)$ for : 6
 (i) $z = (1 - i)(6 + 6i)$
 (ii) $z = (7 - 7\sqrt{3}i)(-1 - i)$.
- (c) Solve the equation : 6
 $z^4 = 5(z - 1)(z^2 - z + 1)$.
- (a) For $a, b \in \mathbf{Z}$, define $a \sim b$ iff $a^2 - b^2$ is divisible by 3 : 6
 (i) Prove that \sim is an equivalence relation on \mathbf{Z} .
 (ii) Find the equivalence classes of 0 and 1.
- (b) Define : 6
 $f : \mathbf{Z} \rightarrow \mathbf{Z}$ by $f(x) = x^2 - 5x + 5$
 (i) Is f one-to-one ?
 (ii) Is f onto ?
 Justify each answer.

P.T.O.

(c) Show that the open intervals $(0, 1)$ and $(4, 6)$ have the same cardinality.

3. (a) Suppose a , b and c are three non-zero integers with a and c relatively prime. Show that :

$$\gcd(a, bc) = \gcd(a, b).$$

(b) (i) Solve the following congruence if possible. If no solution exists, explain why not :

$$4x \equiv 2 \pmod{6}.$$

(ii) Find three positive and three negative integers $\bar{5}$ w.r.t. congruence mod 7.

(c) Use mathematical induction to establish the following inequality :

$$n! > n^3, \text{ for all } n \geq 6.$$

4. (a) Find the general solution to the following linear system :

$$3x_2 - 6x_3 + 6x_4 + 4x_5 = -5$$

$$3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9$$

$$3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15.$$

(b) Let $u = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$.

Is u in the subspace of \mathbf{R}^3 spanned by the columns of A . Why or why not ?

(c) Let :

$$v_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, v_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}.$$

(i) For what values of h is v_3 in span $\{v_1, v_2\}$?

(ii) For what values of h is $\{v_1, v_2, v_3\}$ linearly dependent ? Justify each answer. $6\frac{1}{2}$

5. (a) Let $A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 1 \\ 3 & 6 & 0 \end{bmatrix}$, and define by $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by

$T(x) = Ax$. Find all x in \mathbb{R}^3 such that $T(x) = 0$. Does

$b = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ belong to range of T ? $6\frac{1}{2}$

(b) A linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ first reflects points through the x_1 -axis and then reflects points through the x_2 -axis. Show that T can also be described as a linear transformation that rotates points about the origin. What is the angle of that rotation ? $6\frac{1}{2}$

(c) Let

$$A = \begin{bmatrix} 2 & -3 & -4 \\ -8 & 8 & 6 \\ 6 & -7 & -7 \end{bmatrix} \text{ and } u = \begin{bmatrix} 6 \\ -10 \\ 11 \end{bmatrix}.$$

Is u in Nul A ? Is u in Col A ? Justify each answer. $6\frac{1}{2}$

P.T.O.

6. (a) Given $b_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $b_2 = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$ and $B = \{b_1, b_2\}$ is basis of subspace H of \mathbb{R}^2 .

(i) Determine if $x = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ belongs to H .

(ii) Find $[x]_B$, the B -coordinate vector of x . 6½

- (b) Determine the basis of the null space of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & -5 & 0 & -1 \\ 2 & 5 & -8 & 4 & 3 \\ -3 & -9 & 9 & -7 & -2 \\ 3 & 10 & -7 & 11 & 7 \end{bmatrix}$$

- (c) Is $\lambda = -2$ an eigenvalue of $\begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$.

If so, find one corresponding eigenvector. 6½